

The fate of carbon-based life as a function of the light quark mass

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The Hoyle state plays a crucial role in the helium burning of stars that have reached the red giant stage. The close proximity of this state to the triple-alpha threshold is needed for the production of carbon, oxygen, and other elements necessary for life. We investigate whether this life-essential condition is robust or delicately fine-tuned by measuring its dependence on the fundamental constants of nature, specifically the light quark mass and the strength of the electromagnetic interaction. We show that there exist strong correlations between the alpha-particle binding energy and the various energies relevant to the triple-alpha process. We derive limits on the variation of these fundamental parameters from the requirement that sufficient amounts of carbon and oxygen be generated in stars. We also discuss the implications of our results for an anthropic view of the Universe.

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Life as we know it depends on the availability of carbon and oxygen. These two essential elements are produced during helium burning in red giant stars. The initial reaction is the so-called triple-alpha process, where three helium nuclei fuse to generate ^{12}C . This can be viewed as a two-step process. First, two ^4He nuclei combine to form an unstable, but long-lived ^8Be resonance. This ^8Be resonance must then combine with a third alpha-particle to generate carbon. By itself, this process cannot explain the observed abundance of carbon in the Universe. Therefore, Hoyle postulated that a new excited state of ^{12}C , a spinless even-parity resonance near the ^8Be -alpha threshold, enhances the reaction [1]. Soon after this prediction, the new state was found at Caltech [2, 3] and has since been investigated in laboratories worldwide. The measured energy of this second 0^+ state is $\varepsilon = 397.47(18)$ keV above the triple-alpha threshold, while the total and radiative widths are known to be $\Gamma_{\text{tot}} = 8.3(1.0)$ eV and $\Gamma_\gamma = 3.7(5)$ meV, respectively. The reaction rate for the (resonant) triple-alpha process is approximately given by [4]

$$r_{3\alpha} \propto \Gamma_\gamma (N_\alpha/k_B T)^3 \exp(-\varepsilon/k_B T), \quad (1)$$

with N_α the alpha-particle number density, T the stellar temperature and k_B Boltzmann's constant. Due to the exponential dependence, ε is the dominant control parameter of this reaction. Here, we study the dependence of ε upon the fundamental parameters of the strong and electromagnetic (EM) interactions.

Given its role in the formation of life-essential elements, the Hoyle state has been called the “level of life” [5] (see Ref. [6] for a thorough discussion of the history of this issue). Thus, it is often considered a prime example of the anthropic principle, which states that the observed values of the fundamental physical and cosmological parameters are restricted by the requirement that life can form to observe them, and that the current Universe be old enough for that to happen [7, 8]. In the

context of cosmology and string theory, consequences derived from anthropic considerations have had considerable impact (see *e.g.* Refs. [9, 10]).

Several numerical studies have investigated the impact of changes in the Hoyle state energy. Livio *et al.* [11] modified the value of ε by hand and performed calculations involving the triple-alpha process in the core and helium shell burning of helium up to the asymptotic giant branch stage in stellar evolution. They concluded that a $\simeq 60$ keV change in ε could be tolerated, and thus the amount of fine-tuning required was not as severe as first believed.

A more microscopic calculation was performed by Oberhummer *et al.* [4, 12] in terms of a nuclear cluster model based on a simple two-nucleon (NN) + EM interaction. This NN interaction was formulated in terms of one strength parameter, adjusted to give a fair description of α - α scattering and the spectrum of ^{12}C . By modifying this coupling strength and the EM fine structure constant α_{em} , the effect on carbon and oxygen production was analyzed. Outside of a narrow window of $\simeq 0.5\%$ around the observed strong force and $\simeq 4\%$ around the observed Coulomb force, the stellar production of carbon and/or oxygen was found to be reduced by several orders of magnitude. However, this model of the strong force is not readily connected to the fundamental theory of the strong interactions, quantum chromodynamics (QCD), and its fundamental parameters, the light quark masses. Therefore, it is not obvious how to translate the findings of Ref. [12] into anthropic constraints on fundamental parameters. In this study, we shall address this pertinent question: What changes in the quark masses and the EM fine structure constant are consistent with the formation of carbon-based life?

Over the last few years, we have developed a new method to study atomic nuclei and their properties from first principles, termed *nuclear lattice simulations*. The key ingredients in this approach are, on the one hand, the chiral effective field

theory (EFT) of nuclear forces and, on the other hand, large-scale lattice Monte Carlo methods. The latter are also fruitfully used in many other fields of science. Chiral nuclear EFT was introduced by Weinberg [13] (for a first numerical implementation, see [14]) as a systematic tool to explore the consequences of spontaneous and explicit chiral symmetry breaking of QCD in a rigorous manner. The basic degrees of freedom are pions and nucleons, where the pions and their interactions carry the basic information of the chiral symmetry properties of QCD. In particular, one finds $M_\pi^2 \sim (m_u + m_d)$, so that any dependence on the light quark masses m_u and m_d can be translated into a corresponding dependence on the pion mass M_π . In what follows, only the average light quark mass $m_q \equiv (m_u + m_d)/2$ will be considered, as the effects of strong isospin violation due to $m_u \neq m_d$ are greatly suppressed for the reactions considered here. Chiral nuclear EFT is based on an order-by-order expansion of the nuclear potential. In this scheme, two-, three- and four-nucleon forces arise naturally, and their observed hierarchy is also explained. The nuclear forces have been worked out to high precision and applied successfully in few-nucleon systems for binding energies, structure, and reactions. For recent reviews, see Refs. [15, 16]. Within chiral nuclear EFT, the quark mass dependence of light nuclei and its impact on big bang nucleosynthesis has already been studied, see *e.g.* Refs. [17–22] and [23] for a related study.

Monte Carlo simulations have been used to solve the nuclear A -body problem (with A the atomic number) based on a lattice formulation [24]. The lattice spacing of this discretized space-time serves as an ultraviolet regulator. The nucleons are placed on the lattice sites, and the interactions are represented by pionic and (suitably chosen) auxiliary fields. Our periodic cubic lattice has a spacing of $a = 1.97$ fm and a length of $L = 11.82$ fm. In the time direction, our lattice spacing is $a_t = 1.32$ fm, and the propagation time L_t is varied in order to extrapolate to $L_t \rightarrow \infty$. The energies of the ground and excited states are obtained using projection Monte Carlo techniques [25, 26]. More precisely, we compute $Z_A(t) \equiv \langle \psi_A | \exp(-Ht) | \psi_A \rangle$ for a given A -nucleon system at large Euclidean time t in order to extract the energies of the low-lying states, see also Ref. [27] for more details.

The leading order (LO) contribution to the NN force emerges from the one-pion exchange potential (OPEP) and (smeared) S -wave contact interactions. This improved LO action forms the basis of our projection Monte Carlo simulations, while all higher-order terms including the Coulomb interaction, corrections to the NN force and three-nucleon forces, are treated in perturbation theory. All parameters of H are fixed from two- and three-nucleon data, enabling predictions for all heavier nuclei. So far, such calculations have been performed up to next-to-next-to-leading order (NNLO), achieving a good description of nuclei up to $A = 12$.

We have performed the first *ab initio* calculations for the energy [25] and structure of the Hoyle state [26] using the nuclear lattice formalism (see Ref. [28] for a no-core shell model calculation of the spectrum of ^{12}C employing chiral

EFT forces). In our approach, the hadronic interactions of the nucleons with themselves and with pions can be modified easily. Our analysis of the dependence upon the strength of the Coulomb interaction is therefore straightforward. For the dependence on m_q , we also need information about the quark mass dependence of the hadronic interactions. In turn, such dependences can be given as a function of M_π .

We shall restrict ourselves to values of M_π near the physical point, with $|\delta M_\pi/M_\pi| \leq 10\%$. Such small changes can be treated in perturbation theory. The M_π -dependence of the OPEP and the nucleon mass m_N is determined in chiral perturbation theory utilizing constraints from lattice QCD, see Ref. [31] for more details. To retain model independence, we do not rely on the chiral expansion of the NN contact interactions. Instead, we express our results in terms of the derivatives of the inverse spin-singlet and spin-triplet NN scattering lengths with respect to the pion mass,

$$\bar{A}_s \equiv \left. \frac{\partial a_s^{-1}}{\partial M_\pi} \right|_{M_\pi^{\text{ph}}}, \quad \bar{A}_t \equiv \left. \frac{\partial a_t^{-1}}{\partial M_\pi} \right|_{M_\pi^{\text{ph}}}, \quad (2)$$

which parameterize the M_π -dependence of the short-range nuclear force and can be measured in lattice QCD. We do not consider M_π -dependent short-range effects beyond the ones introduced above.¹ Thus, the variation of a given nuclear energy level E_i takes the form

$$\begin{aligned} \left. \frac{\partial E_i}{\partial M_\pi} \right|_{M_\pi^{\text{ph}}} &= \left. \frac{\partial E_i}{\partial M_\pi^{\text{OPE}}} \right|_{M_\pi^{\text{ph}}} + x_1 \left. \frac{\partial E_i}{\partial m_N} \right|_{m_N^{\text{ph}}} \\ &+ x_2 \left. \frac{\partial E_i}{\partial \tilde{g}_{\pi N}} \right|_{\tilde{g}_{\pi N}^{\text{ph}}} + x_3 \left. \frac{\partial E_i}{\partial C_0} \right|_{C_0^{\text{ph}}} + x_4 \left. \frac{\partial E_i}{\partial C_I} \right|_{C_I^{\text{ph}}}, \end{aligned} \quad (3)$$

with $x_1 \equiv \partial m_N / \partial M_\pi|_{M_\pi^{\text{ph}}}$, $x_2 \equiv \partial \tilde{g}_{\pi N} / \partial M_\pi|_{M_\pi^{\text{ph}}}$ *etc.*, where X^{ph} denotes the value of X for the physical M_π . The terms in Eq. (3) represent different contributions to the pion mass variation. First, there is the explicit dependence on M_π through the pion propagator in the OPEP. Second, we include the dependences on M_π through the nucleon mass m_N and $\tilde{g}_{\pi N} \equiv g_A/(2F_\pi)$, with g_A the nucleon axial-vector coupling and F_π the weak pion decay constant. Finally, we have the M_π -dependences from the strengths of the NN contact interactions C_0 and C_I , which are expressed through the derivatives given in Eq.(2). Therefore, the problem reduces to the calculation of various derivatives of the nuclear energy levels using lattice Monte Carlo techniques and the determination of the coefficients $x_1 \dots x_4$. The derivatives of E_i in Eq. (3) are computed by evaluating the expectation value of the derivative of the lattice Hamiltonian H with respect to M_π^{OPE} , m_N , $\tilde{g}_{\pi N}$, C_0 and C_I . This involved the generation of $\mathcal{O}(10^7)$ statistically independent pion- and auxiliary field configurations

¹ The correlations observed for various energy differences (as discussed below) indicate that the dynamics of interest are largely governed by the large S -wave NN scattering lengths. Higher-order M_π -dependent short-range terms are therefore expected to play a minor role.

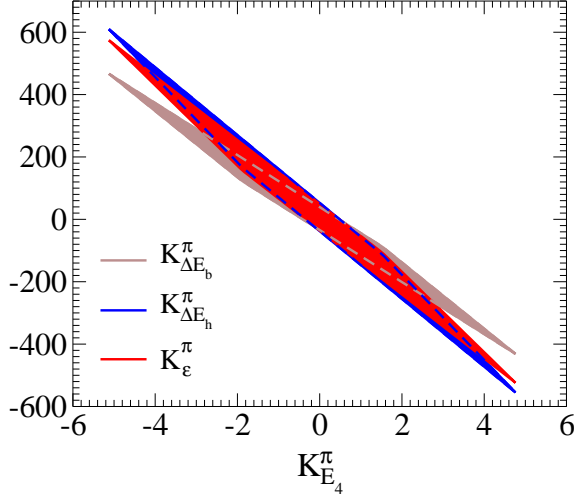


FIG. 1: Correlation of the α -particle energy E_4 with the energy differences pertinent to the triple-alpha process under variation of $\bar{A}_{s,t}$ in the range $\{-1 \dots 1\}$.

on the Blue Gene/Q supercomputer JUQUEEN using the hybrid Monte Carlo algorithm. The explicit form of H can be found in Ref. [29].

The values of x_1 and x_2 can be obtained from lattice QCD combined with chiral extrapolations (see *e.g.* Ref. [30] for a recent review on lattice QCD and determinations of the nucleon mass variation). We exchange x_3 and x_4 for \bar{A}_s and \bar{A}_t by consideration of the M_π -dependence of NN scattering in a cubic box. We may then compute the energy differences $\Delta E_h \equiv E_{12}^* - E_8 - E_4$ and $\Delta E_b \equiv E_8 - 2E_4$, where E_{12}^* is the energy of the Hoyle state and $E_{4,8}$ the ground-state energies of the ${}^4\text{He}$ and ${}^8\text{Be}$ nuclei, respectively. Note also that $\epsilon \equiv \Delta E_h + \Delta E_b$. We find

$$\begin{aligned} \left. \frac{\partial \Delta E_h}{\partial M_\pi} \right|_{M_\pi^{\text{ph}}} &= -0.455(35)\bar{A}_s - 0.744(24)\bar{A}_t + 0.051(19), \\ \left. \frac{\partial \Delta E_b}{\partial M_\pi} \right|_{M_\pi^{\text{ph}}} &= -0.117(34)\bar{A}_s - 0.189(24)\bar{A}_t + 0.013(12), \\ \left. \frac{\partial \epsilon}{\partial M_\pi} \right|_{M_\pi^{\text{ph}}} &= -0.572(19)\bar{A}_s - 0.933(15)\bar{A}_t + 0.064(16), \end{aligned} \quad (4)$$

where the parentheses represent the one-standard-deviation stochastic (Monte Carlo) and extrapolation error, combined with the uncertainty in $x_{1,2}$ (as explained in Ref. [31]) which affects only the constant (OPEP) terms above.

The results in Eq. (4) are intriguing. First, we note that $(\partial \Delta E_h / \partial M_\pi) / (\partial \Delta E_b / \partial M_\pi) \simeq 4$, thus ΔE_h and ΔE_b cannot be independently fine-tuned. Such behavior can readily be explained in terms of the α -cluster structure of the Hoyle state and ${}^8\text{Be}$. Further correlations are visualized in Fig. 1, where the relative changes in ΔE_b , ΔE_h and ϵ are shown as a function of relative changes in the ground state energy E_4 of the α -particle. We define $K_X^\pi \equiv (\partial X / \partial M_\pi) M_\pi / X$ as the relative variation of X with respect to M_π . Fig. 1 provides

clear evidence that the alpha binding energy is strongly correlated with ΔE_b , ΔE_h , and ϵ . Such correlations related to carbon production have been speculated upon earlier [11, 32]. Second, we note that there is a special value for the ratio of \bar{A}_s to \bar{A}_t , given by

$$\bar{A}_s / \bar{A}_t \simeq -1.5, \quad (5)$$

where the pion mass dependence of ΔE_h , ΔE_b , and ϵ becomes small (compared to the error bars).

We have expressed all our results in terms of the quantities $\bar{A}_{s,t}$, the quark mass dependence of which was considered at next-to-leading order (NLO) in Ref. [18], with a recent update to NNLO [31]. That analysis gives $\bar{A}_s = 0.29^{+0.25}_{-0.23}$ and $\bar{A}_t = -0.18^{+0.10}_{-0.10}$, where the errors reflect the theoretical uncertainties. As expected, these values of $\bar{A}_{s,t}$ are of natural size. Taking into account correlations in the calculation of $\bar{A}_{s,t}$, we find $\bar{A}_s / \bar{A}_t = -1.6^{+1.0}_{-1.7}$. Interestingly, the central value is very close to the result given in Eq. (5), for which the pion mass dependences of ΔE_h , ΔE_b , and ϵ are all approximately zero (within error bars). In the future, a reduction of the uncertainty in $\bar{A}_{s,t}$ is desirable. This can be addressed by lattice QCD calculations of NN systems. For recent studies, see Refs. [33, 34].

We now use the reaction rate in Eq. (1) to draw conclusions about the allowed variations of the fundamental constants. From the stellar modeling calculations in Ref. [12], we find that sufficient abundances of both carbon and oxygen can be maintained within an envelope of ± 100 keV around the observed value of ϵ . Allowing for a maximum shift of ± 100 keV in ϵ translates into bounds on the variations of m_q . In Fig. 2, we show “survivability bands” for carbon-oxygen based life due to 1% and 5% changes in m_q (in terms of \bar{A}_s and \bar{A}_t). To be precise, for a 5% change in m_q , \bar{A}_t must assume values within the red (narrow) band to allow for sufficient production of carbon and oxygen. The most up-to-date knowledge of these parameters is depicted by the data point with horizontal and vertical error bars. This NNLO determination of $\bar{A}_{s,t}$ shows that carbon-based life survives at least a $\simeq 0.7\%$ shift in m_q . In addition to this “worst-case scenario”, we find that the theoretical uncertainty in $\bar{A}_{s,t}$ is also compatible with a vanishing $\partial \epsilon / \partial M_\pi$ (complete lack of fine-tuning). Given the central values of $\bar{A}_{s,t}$, we conclude that variations of the light quark masses of 2 – 3% are unlikely to be catastrophic to the formation of life-essential carbon and oxygen.

We may also compute the corresponding changes induced by variations of the EM fine-structure constant α_{em} . On the lattice, the EM shift receives contributions from the long-range Coulomb force and a short-range proton-proton contact interaction. The latter contains an unknown coupling strength, which allows for the regularization of QED on the lattice. We have fixed its finite part from the known EM contribution to the α -particle binding energy. The dependence of the E_i on α_{em} can then be calculated. By expressing the EM shifts as $(\partial X / \partial \alpha_{\text{em}})|_{\alpha_{\text{em}}^{\text{ph}}} \simeq Q(X) / \alpha_{\text{em}}$, we find $Q(\Delta E_b) = 1.19(8)$ MeV, $Q(\Delta E_h) = 2.80(10)$ MeV and

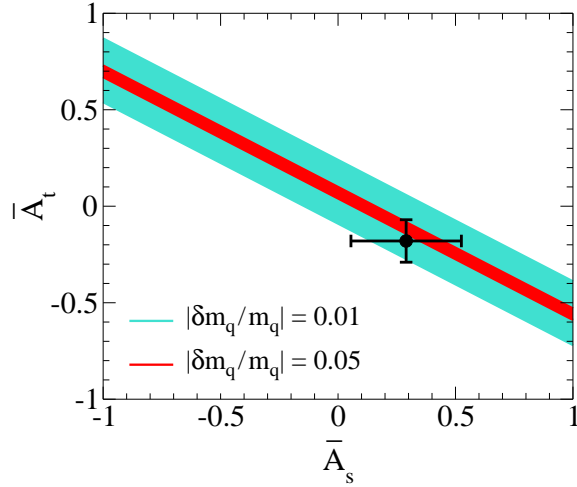


FIG. 2: “Survivability bands” for carbon-oxygen based life from Eq. (4), due to 1% and 5% changes in m_q in terms of the parameters \bar{A}_s and \bar{A}_t . The most up-to-date NNLO analysis of $\bar{A}_{s,t}$ is depicted by the data point with horizontal and vertical error bars.

$Q(\varepsilon) = 3.99(9)$ MeV. For fixed m_q , a variation of α_{em} by $\pm 100 \text{ keV}/Q(\varepsilon) \approx 2.5\%$ would thus be compatible with the formation of carbon and oxygen in our Universe. This is consistent with the $\simeq 4\%$ bound reported in Ref. [4].

In summary, we have presented *ab initio* lattice calculations of the dependence of the triple-alpha process upon the light quark masses and the EM fine structure constant. The position of the ^8Be ground state relative to the $2\text{--}\alpha$ threshold, as well as that of the Hoyle state relative to the $3\text{--}\alpha$ threshold, appears strongly correlated with the binding energy of the α -particle. We also find that the formation of carbon and oxygen in our Universe would survive a change of $\simeq 2\%$ in m_q or $\simeq 2\%$ in α_{em} . Beyond such relatively small changes, the anthropic principle appears necessary at this time to explain the observed reaction rate of the triple-alpha process. In order to make more definitive statements about carbon and oxygen production for larger changes in the fundamental parameters, a more precise determination of \bar{A}_s and \bar{A}_t is needed from future lattice QCD simulations.

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